
Trajectory Graph Learning: Aligning with Long Trajectories in Reinforcement Learning Without Reward Design

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Abstract

Reinforcement learning (RL) often relies on manually designed reward functions, which are difficult to specify and can lead to issues such as reward hacking and suboptimal behavior. Alternatives like inverse RL and preference-based RL attempt to infer surrogate rewards from demonstrations or preferences but suffer from ambiguity and distribution mismatch. A more direct approach, inspired by imitation learning, avoids reward modeling by leveraging expert demonstrations. However, most existing methods align actions only at individual states, failing to capture the coherence of long-horizon trajectories.

In this work, we study the problem of directly aligning policies with expert-labeled trajectories to preserve long-horizon behavior without relying on reward signals. Specifically, we aim to learn a policy that maximizes the probability of generating the expert trajectories. Nevertheless, we prove that, in its general form, this trajectory alignment problem is NP-complete. To address this, we propose *Trajectory Graph Learning* (TGL), a framework that leverages structural assumptions commonly satisfied in practice—such as bounded realizability of expert trajectories or a tree-structured MDP. These enable a graph-based policy planning algorithm that computes optimal policies in polynomial time under known dynamics. For settings with unknown dynamics, we develop a sample-efficient algorithm based on UCB-style exploration and establish sub-linear regret. Experiments on grid-world tasks demonstrate that TGL substantially outperforms standard imitation learning methods for long-trajectory planning.

1 Introduction

Reinforcement learning (RL) has emerged as a powerful tool with numerous successful applications, ranging from early advancements in robotics and control [Lee et al., 2020] to more recent breakthroughs in self-driving technology [Dosovitskiy et al., 2017] and the fine-tuning of Large Language Models (LLMs) [Ouyang et al., 2022]. However, the success of RL in real-world applications often relies heavily on the design of the reward function, which typically requires significant prior knowledge. We face challenges such as reward hacking [Amodei et al., 2016] where unintended behaviors maximize the given reward and reward shaping [Ng et al., 1999] where improperly crafted rewards lead to suboptimal learning.

Since designing scalar, numeric rewards is often impractical for complex real-world tasks, some alternative paradigms have been proposed. Inverse Reinforcement Learning (IRL) [Ng et al., 2000] aims to recover a reward function that explains and justifies an expert’s demonstrated

behavior—so the agent can reproduce comparable policies without ever being given explicit rewards. More recently, Preference-based Reinforcement Learning (PbRL) [Akrou et al., 2011b, Christiano et al., 2017, Xu et al., 2020, Abdelkareem et al., 2022] replaces handcrafted numeric rewards with human preferences—typically pairwise or ordinal feedback over trajectories or behaviors—and learns either a surrogate reward or a policy that aligns with those preferences to guide decision-making. However, learning a surrogate reward from demonstrations or preferences does not guarantee alignment with expert intent; ambiguity [Wagh et al., 2013, Lambert and Calandra, 2023, Hu et al., 2023], overfitting [Brown et al., 2019a, Szot et al., 2023] and distribution shift [Fu et al., 2017] all conspire to degrade final policy quality. These empirical and theoretical findings motivate approaches—such as direct trajectory alignment that bypass reward modelling altogether.

Imitation learning [Hussein et al., 2017] circumvents the need for handcrafted reward functions by training policies to replicate expert behavior, typically by learning a mapping from observed states to corresponding expert actions. However, its classical instantiation—behavior cloning (BC) [Torabi et al., 2018]—focuses solely on *state-action pair* alignment, capturing the expert action conditioned on individual states while neglecting the broader trajectory-level structure. As a result, discrepancies between the trajectories generated by the learned policy and the expert demonstrations can accumulate over time, ultimately undermining the preservation of coherent long-horizon behaviors [Ross et al., 2011, Chang et al., 2021]. We illustrate this limitation with a concrete example in Appendix A.2, where BC fails to reliably reproduce entire expert trajectories with probability 1 even in a deterministic environment. This shortcoming motivates our study of *direct trajectory alignment*, which seeks to directly maximize the likelihood of reproducing complete expert trajectories. Such alignment is particularly critical in applications like large language model (LLM) response generation [Zeng et al., 2024] and autonomous driving [Huang et al., 2024], where even small local deviations can propagate into significant degradations in overall quality or safety. To the best of our knowledge, the theoretical foundations of directly aligning policies with expert-labeled trajectories—without relying on reward modeling—remain largely unexplored.

In this paper, we systematically investigate the problem of direct trajectory alignment in reinforcement learning and develop a theoretical framework that enables efficient policy learning through *whole-trajectory* alignment. Our approach eliminates the need for explicit reward modeling and leverages structural assumptions such as reachable expert trajectories in terms of probability and tree-structured MDP to ensure computational tractability. We summarize our key contributions below.

Our Contributions

- **Hardness result for direct trajectory alignment.** We prove that the general problem of finding an optimal policy via direct trajectory alignment is NP-complete, by presenting a novel reduction from a classical NP-complete problem. This result highlights the fundamental computational challenge of directly aligning policies with expert trajectories.
- **Theoretical framework: *Trajectory Graph Learning* (TGL).** We introduce *Trajectory Graph Learning* (TGL), a theoretical framework that casts the direct trajectory alignment problem as a maximum weight independent set problem over a trajectory-induced graph. Under structural assumptions—such as bounded realizability of expert trajectories or a tree-structured MDP—we show that the optimal policy can be computed in polynomial time. In the setting with unknown dynamics, we integrate an *upper-confidence bound* (UCB) exploration strategy and design a learning algorithm with provably sub-linear cumulative regret.
- **Empirical validation.** We empirically evaluate TGL on grid-world benchmarks. Our results show that TGL consistently aligns with expert-labeled trajectories more faithfully than standard behavior cloning across various trajectory sets.

Related work We review several classical RL approaches. The first line of work is inverse reinforcement learning (IRL), which infers surrogate rewards from expert demonstrations; the second is preference-based RL (PbRL), which learns from preferences over trajectories; and the third is imitation learning, which directly maps expert demonstrations to policies without reward modeling.

Inverse RL Early work framed IRL as reward reconstruction: Ng et al. [2000]’s linear program formulation and Abbeel and Ng [2004]’s feature expectation matching seek a reward under which the expert is optimal, a paradigm later disambiguated with maximum-entropy regularization [Ziebart et al., 2008]. Deep variants such as Guided Cost Learning [Finn et al., 2016], GAIL [Ho and Ermon, 2016], and AIRL [Fu et al., 2018] scale this two-stage pipeline, yet their rewards remain trustworthy only near the demonstration distribution, leading to policy mis-alignment after exploration. Recent extrapolation and theory papers such as T-REX [Brown et al., 2019b], the finite-sample analysis [Komanduru and Honorio, 2019], and empirical audits of RLHF reward models [Kaplan and et al., 2023] confirm that surrogate rewards do not guarantee alignment, motivating the need to analyze direct trajectory alignment as an alternative.

PbRL From the first simulator-free preference-based policy learning algorithms [Akrou et al., 2011a, Wilson et al., 2012, Busa-Fekete et al., 2013] to the widely cited deep RL from human preferences framework [Christiano et al., 2017], most PbRL methods fit a surrogate reward to pairwise or K -wise feedback and then optimize it with standard RL. Recent PbRL algorithms still fit *implicit* reward surrogates whose quality hinges on pre-chosen feature embeddings and Bradley–Terry–Luce (BTL) style preference models. For instance, Saha et al. [2023] established finite-time regret bounds only after projecting trajectories into a hand-crafted feature space, so alignment requires that this embedding fully captures task-relevant differences. The finite-sample analysis of Xu et al. [2020] likewise assumes an unobserved latent reward and proves guarantees under the oracle condition that pairwise labels reflect that hidden function. Empirical pipelines such as Direct Preference Optimisation (DPO) [Rafailov et al., 2023] and Direct PB-PO [An et al., 2023] optimize the same BTL-based surrogate, while more recent theory—SeqRank’s principled comparison loss [Zhu et al., 2023], the reward-agnostic RAPT optimiser [Zhan et al., 2023], and best-policy identification from preferences [Agnihotri et al., 2025]—all rely on accurate trajectory embeddings and preference-likelihood calibration to recover near-optimal policies. These dependencies indicate that PbRL, which relies on feature embeddings and preference models, may fail to guarantee alignment with expert behavior—highlighting the need to study the problem of direct trajectory alignment.

Imitation Learning Imitation learning (IL) dispenses with reward design by training a policy to copy expert demonstrations [Hussein et al., 2017]. The dominant variant, behaviour cloning, maps each observed state to the expert’s action [Torabi et al., 2018]; because it matches actions *state-by-state*, it incurs covariate shift, so small errors push the learner into unseen states and the mismatch compounds along a trajectory [Ross et al., 2011]. Recent analyses confirm that even with offline fixes such as MILO trajectory-level fidelity remains loosely bounded [Chang et al., 2021]. Thus, no existing IL framework directly optimizes expected alignment with a set of expert-labelled trajectories.

2 Problem Formulation

In this section, we first introduce and define the direct trajectory alignment problem. We consider a finite-horizon Markov Decision Process (MDP) $M = (\mathcal{S}, \mathcal{A}, \mathbb{P}, \mathcal{T}_{\text{sel}}, H, \mu)$, where \mathcal{S} is the state space, \mathcal{A} is a finite action space, $\mathbb{P} : \mathcal{S} \times \mathcal{A} \rightarrow \Delta(\mathcal{S})$ is the transition kernel, H is the planning horizon (i.e., episode length), and μ is the initial state distribution. The agent interacts with the environment episodically. In each episode of length H , the agent follows a policy $\pi = \{\pi_h\}_{h=1}^H$, where each $\pi_h : \mathcal{S} \rightarrow \mathcal{A}$ maps a state to an action at time step $h \in [H]$. A policy π induces a trajectory $\tau = (s_1, a_1, s_2, a_2, \dots, s_H, a_H)$, where $s_1 \sim \mu$, $a_1 = \pi_1(s_1)$, $s_2 \sim \mathbb{P}(\cdot | s_1, a_1)$, $a_2 = \pi_2(s_2)$, and so on. Let \mathcal{T} denote the set of all possible H -length trajectories in the environment. Instead of having a reward function as in traditional RL, we are given a *Expert-Labeled Trajectory Set*, defined as a small subset $\mathcal{T}_{\text{sel}} = \{\tau_i\}_{i=1}^M \subset \mathcal{T}$ ¹, where each τ_i is a trajectory. Typically, the size of the expert-labeled trajectory set is small relative to the entire trajectory space, i.e., $M \ll |\mathcal{T}|$.

¹Our setting can be naturally extended to scenarios where the expert-labeled trajectory set is accompanied by scalar feedback, such as scores provided by experts. In this case, we have $\mathcal{T}_{\text{sel}} = \{(\tau_i, y_i)\}_{i=1}^M \subset \mathcal{T} \times \mathcal{Y}$, where each τ_i denotes a trajectory and $y_i \in \mathcal{Y}$ represents its associated scalar label. Accordingly, the objective should incorporate the feedback by weighting each trajectory with its corresponding label y_i .

Each policy π induces a distribution d^π over the space of H -length trajectories \mathcal{T} , where $d^\pi(\tau) = \mu(s_1) \prod_{h=1}^H \pi(a_h | s_h) \cdot \mathbb{P}(s_{h+1} | s_h, a_h)$. The objective is to find a policy that maximizes the visitation probability of the trajectories in the expert-labeled set.

$$\pi^* = \arg \max_{\pi \in \Pi} \sum_{\tau \in \mathcal{T}_{\text{sel}}} d^\pi(\tau)$$

Direct trajectory alignment replaces the cumulative reward used in standard RL with an expert-labeled trajectory set as the criterion for policy evaluation. In standard RL, the effectiveness of a learning algorithm is often measured by *regret*, defined as the difference between the cumulative reward of the optimal policy and that of the agent's learned policy over time. Analogously, in our direct trajectory alignment setting, we can define a notion of regret that captures the performance gap in aligning with the expert-labeled trajectories. Specifically, for each round $k \in [K]$, suppose the agent starts from the same initial distribution μ and executes policy π^k to generate a trajectory. The cumulative regret after K rounds is then defined as

$$\text{Regret}(K) = \sum_{k=1}^K \left(\max_{\pi \in \Pi} \sum_{\tau \in \mathcal{T}_{\text{sel}}} d^\pi(\tau) - \sum_{\tau \in \mathcal{T}_{\text{sel}}} d^{\pi^k}(\tau) \right), \quad (1)$$

where $d^\pi(\tau)$ denotes the probability of generating trajectory τ under policy π .

Notation We use $[n]$ to represent index set $\{1, \dots, n\}$. For $x \in \mathbb{R}$, $\lfloor x \rfloor$ represents the largest integer not exceeding x and $\lceil x \rceil$ represents the smallest integer exceeding x . We use O to represent leading orders in asymptotic upper bounds and \tilde{O} to hide the polylog factors. For a finite set \mathcal{A} , we denote the cardinality of \mathcal{A} by $|\mathcal{A}|$.

3 Hardness of General Direct Trajectory Alignment Problem

In this section, we will show the hardness of solving a general direct trajectory alignment problem. We will show that getting the optimal policy in a general direct trajectory alignment problem is equivalent to solving a **Maximum-Weight Independent Set (MWIS)** problem in a graph. First, we will introduce some concepts here.

Definition 1 (Conflict in State-Action Pair). *Two state-action pairs (s, a) and (s', a') are in conflict if they share the same state but have different actions:*

$$s = s' \quad \text{and} \quad a \neq a'.$$

Definition 2 (Conflict in Trajectories). *Consider two trajectories of length H :*

$$\tau_1 = (s_1, a_1, \dots, s_H, a_H), \quad \tau_2 = (s'_1, a'_1, \dots, s'_H, a'_H).$$

They are in conflict if there exists $h \in \{1, \dots, H\}$ such that

$$s_h = s'_h \quad \text{and} \quad a_h \neq a'_h.$$

Remark. *If τ_1 and τ_2 conflict, and τ_1 and τ_3 do not conflict, it does not imply that τ_2 and τ_3 are conflict-free.*

Definition 3 (Trajectory-Induced Policy Set). *For a trajectory $\tau = (s_1, a_1, \dots, s_H, a_H)$, its trajectory-induced policy set π^τ contains all policies $\pi = \{\pi_h\}_{h=1}^H$ satisfying $\pi_h(s_h) = a_h$ for each h , while $\pi_h(s')$ is arbitrary for $s' \neq s_h$:*

$$\pi^\tau = \{\pi \mid \pi_h(s_h) = a_h \ \forall h = 1, \dots, H; \ \pi_h(s') \text{ arbitrary for } s' \neq s_h\}.$$

This set represents all policies that exactly reproduce τ on its specific states. Since we focus on deterministic policies, conflicts arise when a policy must choose a unique action at the same state, unlike randomized policies which can mix actions.

Remark. *If τ_1 and τ_2 conflict, then $\pi^{\tau_1} \cap \pi^{\tau_2} = \emptyset$ because a deterministic policy cannot choose conflicting actions at the same state. Conversely, if a set of trajectories $\{\tau_1, \dots, \tau_m\}$ are pairwise conflict-free, then $\bigcap_{i=1}^m \pi^{\tau_i} \neq \emptyset$.*

Algorithm 1 TGL-CP

Require: Finite-horizon MDP $(\mathcal{S}, \mathcal{A}, P, H)$; Expert-Labeled Trajectory Set $\mathcal{T}_{\text{sel}} = \{\tau_1, \dots, \tau_M\}$; **MWIS oracle** $\text{MWIS}(G, w) \rightarrow S$

Ensure: Chosen trajectory subset S and derived policy π

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/* Conflict graph */
1:  $V \leftarrow \{v_i \mid \tau_i \in \mathcal{T}_{\text{sel}}\}, E \leftarrow \emptyset$ 
2: for all  $(\tau_i, \tau_j)$  with  $i < j$  do
3:   if  $\exists h: s_h^i = s_h^j \wedge a_h^i \neq a_h^j$  then
4:      $E \leftarrow E \cup \{(v_i, v_j)\}$ 

/* Weights */
5: for all  $\tau_i \in \mathcal{T}_{\text{sel}}$  do
6:    $p_i \leftarrow \prod_{h=1}^{H-1} P(s_{h+1}^i \mid s_h^i, a_h^i)$ 

/*MWIS Oracle */
7:  $S \leftarrow \text{MWIS}(G, (p_1, \dots, p_M))$ 
8: for all  $v_i \in S$  with  $\tau_i = (s_1^i, a_1^i, \dots, s_H^i, a_H^i)$  do
9:   for  $h = 1$  to  $H$  do
10:     $\pi_h(s_h^i) \leftarrow a_h^i$ 
11: For states not covered by  $\bigcup_{v_i \in S} \tau_i$ , set  $\pi_h$  via a default rule
12: return  $S, \pi$ 

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182 For each trajectory $\tau_i = (s_1^{(i)}, a_1^{(i)}, \dots, s_H^{(i)}, a_H^{(i)})$, let

$$p_i = \mu(s_1^{(i)}) \cdot P_1(s_2^{(i)} \mid s_1^{(i)}, a_1^{(i)}) \cdots P_{H-1}(s_H^{(i)} \mid s_{H-1}^{(i)}, a_{H-1}^{(i)}).$$

183 If a deterministic policy π belongs to $\bigcap_{i=1}^k \pi^{\tau_i}$, then

$$P(\text{Agent visits } \{\tau_1, \dots, \tau_k\} \mid \pi) = \sum_{i=1}^k p_i.$$

184 At the same time we can consider a **Conflict Graph** $G = (V, E, W)$: where the vertex set
185 V represents the trajectories $\tau_1, \tau_2, \dots, \tau_M$ in \mathcal{T}_{sel} , the edge set E is constructed if any pair
186 of trajectories are conflict. For the weight set $W = \{w_i\}$, we let $w_i = p_i$, the probability
187 product of realizing that trajectory.

188 Thus, the original problem reduces to the following **Maximum-Weight Independent Set**
189 **(MWIS)** problem on graph G with weights p_i :

$$\max \sum_{i=1}^k p_i x_i, \quad x_i + x_j \leq 1 \quad \forall (v_i, v_j) \in E, \quad x_i \in \{0, 1\} \quad \forall i. \quad (2)$$

190 Let $S = \{v_i : x_i = 1\}$ be the optimal node set; the corresponding policy selects $\pi_h(s_h^{(i)}) = a_h^{(i)}$
191 for all $v_i \in S$ and $h = 1, \dots, H$.

192 It is well known that the **Maximum Weight Independent Set (MWIS)** problem in a
193 graph is NP-complete [Garey and Johnson, 1979]. We have demonstrated that our original
194 problem—finding the optimal policy in the binary-labeled setting—can be reduced to an
195 instance of the **MWIS** problem. This motivates us to explore whether these two problems
196 are equivalent in computational complexity. In fact, we establish the following result:

197 **Theorem 1.** *The problem of finding the optimal policy in the direct trajectory alignment*
198 *setting is NP-complete.*

199 **Remark.** *We prove this by showing reduction from a known NP-complete problem –the*
200 *MWIS problem. We show that any weighted graph can be represented as a subset of*
201 *trajectories with corresponding probabilities. We achieve this by using a novel BFS-Based*
202 *trajectory construction to transfer any weighted graph to a subset of trajectories in one MDP.*
203 *Then we show that good policy implies high weight independent set, which concludes the proof.*
204 *The details is provided in the Appendix A.3.*

4 Trajectory Graph Learning with Known Model

When we face the hardness of finding the exact optimal solution of the general problem in polynomial time implied by Theorem 1, one may think of the path to find approximation solution with polynomial time. Unfortunately, prior work by Johan [1999] shows that the MWIS problem is extremely hard to approximate, establishing that unless $P = NP$, there is no $\frac{1}{n^{1-\varepsilon}}$ -approximation algorithm for MWIS for any fixed $\varepsilon > 0$, where n denotes the number of nodes in the graph. However, we establish the *Trajectory Graph Learning* (TGL) framework with positive results when certain assumptions is added on the MDP or the expert-labeled trajectory set.

Case 1: Bounded realizability of expert trajectory set In the first case, we assume that all trajectories in \mathcal{T}_{sel} are reachable, meaning they have non-negligible probability under the environment. This is a standard and practical assumption, as trajectories with vanishingly small probability are often ignored or excluded during data collection or preprocessing. As a result, the selected set \mathcal{T}_{sel} contains only trajectories that the agent has a reasonable chance of encountering.

Definition 4 (ε - Realizable). *We say a trajectory $\tau = (s_1, a_1, s_2, a_2, \dots, s_H, a_H)$ is ε -Realizable if the probability product $P_1(s_2|s_1, a_1) \cdot P_2(s_3|s_2, a_2) \cdots P_{H-1}(s_H|s_{H-1}, a_{H-1}) \geq \varepsilon$.*

Example. In a nearly deterministic environment, where transitions are deterministic or have high probability (e.g., $P(s_{h+1} | s_h, a_h) \geq 0.9$ for all steps), many trajectories are ε -realizable with relatively large ε (e.g., $\varepsilon \approx 0.9^{H-1}$). In contrast, in highly stochastic environments, some trajectories may have exponentially small realization probabilities (e.g., $\sim p^{H-1}$ for small p), making them effectively unreachable in practice unless ε is extremely small.

Then we provide a generic framework **TGL-CP** (*Trajectory Graph Learning-Conflict Planner*), which is also shown in Algorithm 1. Basically, the algorithm will first construct a conflict graph, where the edges can be determined by checking the conflicts in the expert-labeled trajectory set and the weights can be calculated by the probability product of each trajectory. Then, a MWIS oracle is applied to select a subset of trajectories S that forms a solution to the problem. The desired policy is subsequently obtained by following the actions specified in the trajectories contained in S , ensuring the policy aligns with the selected subset of trajectories. To implement this step explicitly, we introduce a simple enumeration-based oracle for MWIS, detailed in Algorithm 3 in Appendix A.1. Then we have the following theorem.

Theorem 2. *Assume that any trajectory in the expert-labeled trajectory set \mathcal{T}_{sel} is ε_0 -Realizable, then after applying Algorithm 1 and 3, it can return the optimal policy π^* with time complexity $O(M^2 H + M^{1/\lceil 1/\varepsilon_0 \rceil})$.*

Remark. *The enumeration-based oracle exploits the lower bound $w(v) \geq \varepsilon_0$ implied by the ε_0 -realizable assumption to deduce that any feasible independent set can contain at most $K = \lfloor 1/\varepsilon_0 \rfloor$ vertices. Leveraging this, the oracle exhaustively enumerates all subsets of vertices of size at most K , checks each for independence, and calculates their total weight. By keeping track of the heaviest feasible subset encountered, the oracle returns the exact maximum-weight independent set in time polynomial in the graph size, assuming K is treated as a constant.*

Case 2: Tree-Structured MDP In the second case, instead of focusing on a general MDP, we instead look for some special structured but very useful MDP setting. The one we will display here is a so-called Tree MDP.

Definition 5 (Tree MDP). *A Tree MDP is defined as follows:*

1. *No subsequent crossover for different states: For any step $h \in [H]$, for any two different states $s_h^{(1)}$ and $s_h^{(2)}$, the subsequent states after these two states should be different, i.e. if the possible visited states after $s_h^{(1)}$ and $s_h^{(2)}$ are $\sigma(s_h^{(1)})$ and $\sigma(s_h^{(2)})$ respectively, then we have $\sigma(s_h^{(1)}) \cap \sigma(s_h^{(2)}) = \emptyset$.*
2. *No subsequent crossover for different actions: For any step $h \in [H]$, each state s_h has possible actions leading to possible successor states with no merges; i.e. if action*

258 a_1 leads to possible states $\sigma^{(1)}(s_h)$ and a different action a_2 leads to $\sigma^{(2)}(s_h)$, then
 259 $\sigma^{(1)}(s_h) \cap \sigma^{(2)}(s_h) = \emptyset$.

260 3. No revisits: once the MDP leaves s_h , it never returns to it in future steps.

261 Consequently, the transition graph is a forward-branching tree. A trajectory is any path
 262 from the initial state s_1 to a layer- H leaf. Two trajectories conflict if at some time h they
 263 coincide in the same state but choose different actions, forming an edge in the conflict graph.

264 Tree MDPs arise naturally in applications where future decisions unfold independently and
 265 paths do not merge. In *LLMs*, decoding strategies like beam search or top- k sampling
 266 generate diverse continuations from a prompt, forming a forward tree where each branch
 267 represents a distinct sequence [AssemblyAI, 2023]. In *self-driving*, planning algorithms
 268 simulate future actions (e.g., turn, accelerate) under constraints that avoid revisiting past
 269 states, resulting in a branching structure of possible trajectories [Zhao et al., 2025].

270 Under the Tree MDP structure, we provide a novel algorithm **TGL-PrunedTree**, which
 271 is detailed in Algorithm 4 in Appendix A.1. It is a backward trajectory selection and
 272 pruning algorithm tailored for solving the policy optimization problem in a finite-horizon
 273 Tree MDP. Given a set of M trajectories $\mathcal{T}_{\text{sel}} = \{\tau_i\}_{i=1}^M$, each of fixed length H , the algorithm
 274 first computes a weight for each trajectory based on the product of transition probabilities
 275 along its path. It then proceeds in a backward fashion, from the final timestep H to the
 276 root, performing aggregation and pruning at each level. For each state s_h at timestep h , it
 277 aggregates the weights of trajectories sharing the same action a_h and keeps only the action
 278 with the highest total weight. This approach prunes conflicting or suboptimal paths and
 279 merges consistent ones, yielding a compatible, high-weights subset of the original trajectories.

280 **Theorem 3.** Under the Tree MDP assumption (Definition 5), Algorithm 4 computes an
 281 optimal policy π^* with time complexity $O(H \cdot M \cdot |A|)$.

282 **Remark.** The full proof is provided in Appendix A.5. The key insight here is that, under
 283 the Tree MDP structure, the optimal policy can be computed efficiently using a linear-time
 284 dynamic programming approach—rather than solving an NP-hard problem as in the general
 285 graph case. This result paves the way for analyzing more efficient algorithms, both in terms
 286 of time and sample complexity, within learning settings that exhibit tree-like structure.

287 5 Trajectory Graph Learning with Unknown Model

288 In real-world scenarios, even when expert trajectories or human-labeled datasets are available,
 289 the underlying environment dynamics are typically unknown—that is, the transition matrix
 290 \mathbb{P} must still be learned. In this section, we investigate methods for jointly learning the
 291 environment and identifying a good policy, a setting commonly referred to as *online learning*.
 292 Specifically, we introduce a UCB-based exploration algorithm and provide its corresponding
 293 regret analysis.

294 In the online learning setting, we propose **TGL-UCB** (Algorithm 2), which learns an optimal
 295 subset of expert-labeled trajectories by combining Monte Carlo sampling with an Upper
 296 Confidence Bound (UCB) exploration strategy [Auer et al., 2002] over a conflict graph. Each
 297 node in the graph represents a trajectory, and edges connect conflicting pairs that cannot be
 298 realized simultaneously. For each trajectory node $v_i \in V$, we track: T_i (the total number of
 299 times a matching policy has been played), N_i (the number of times trajectory v_i has been
 300 realized), and $\hat{\mu}_i = N_i/T_i$ (the empirical realization probability). Initially, for each v_i , we
 301 generate a policy π_i by selecting an arbitrary maximal independent set containing v_i and
 302 play each once to initialize estimates. In each round, TGL-UCB computes optimistic upper
 303 confidence bounds for all trajectories, invokes an MWIS oracle to select an independent
 304 set maximizing the total UCB values, executes the corresponding policy, and updates
 305 estimates. This iterative process balances exploration and exploitation, enabling TGL-UCB
 306 to progressively refine its trajectory selection and align with expert-labeled demonstrations.
 307 Before presenting the main theoretical result, we introduce some essential definitions.

308 For any independent set $S \subset V$, define $p_S = \sum_{v \in S} p_v$ and the optimal value $p^* =$
 309 $\max_{S \text{ indep.}} p_S$. The set of sub-optimal solutions is

$$\mathcal{S}_{\text{sub}} = \{S \subset V \mid p_S < p^*\}.$$

Algorithm 2 TGL-UCB

Require: Expert-Labeled Trajectory Set $\mathcal{T}_{\text{sel}} = \{\tau_1, \dots, \tau_M\}$; Total number of rounds n ;
MWIS oracle $\text{MWIS}(G, w) \rightarrow S$
Ensure: Chosen trajectory subset S and derived policy π

/ Conflict graph */*
1: $V \leftarrow \{v_i \mid \tau_i \in \mathcal{T}_{\text{sel}}\}, E \leftarrow \emptyset$
2: **for all** (τ_i, τ_j) with $i < j$ **do**
3: **if** $\exists h: s_h^i = s_h^j \wedge a_h^i \neq a_h^j$ **then**
4: $E \leftarrow E \cup \{(v_i, v_j)\}$
5: **Initialization:**
6: For each node (trajectory) $v_i \in V$, use variable T_i as the total number of policies played that matches trajectory v_i , variable N_i as the times that v_i is sampled so far, and variable $\hat{\mu}_i$ as the current estimated empirical probability of realizing trajectory τ_i , where $\hat{\mu}_i = \frac{N_i}{T_i}$.
7: For each node $v_i \in V$, select an arbitrary maximum independent $S_i \subset V$ such that $v_i \in S_i$, and get the corresponding policy π_i from S_i .
8: For each $i \in [M]$, play π_i once and update variables T_i and $\hat{\mu}_i$.
9: **for** $t = M + 1, M + 2, \dots, n$ **do**
10: **for** $i \in [M]$ **do**
11: Set $u_i = \hat{\mu}_i + \sqrt{\frac{3 \log t}{2T_i}}$
12: Compute $S^+ \leftarrow \text{MWIS}(G, \{u_v\}_{v \in V})$
13: Play the corresponding policy π^+ from S^+ and update all T_i 's, N_i 's and $\hat{\mu}_i$'s.

310 For each node $v \in V$, define the sub-optimal gaps:

$$\Delta_{\min}^v = p^* - \max \{p_S \mid S \in \mathcal{S}_{\text{sub}}, v \in S\}, \quad \Delta_{\max}^v = p^* - \min \{p_S \mid S \in \mathcal{S}_{\text{sub}}, v \in S\}.$$

311 The global gap bounds are:

$$\Delta_{\min} = \min_{v \in V} \Delta_{\min}^v, \quad \Delta_{\max} = \max_{v \in V} \Delta_{\max}^v.$$

312 **Theorem 4.** *The expected regret of Algorithm 2 over K rounds is at most $O\left(\frac{M^3 \log K \cdot \Delta_{\max}}{\Delta_{\min}^2}\right)$.*

313 **Remark.** *This theorem establishes a gap-dependent regret bound and guarantees sub-linear regret, indicating that efficient learning is achievable in our setting given access to a good*
314 *MWIS oracle. The detailed proof is provided in Appendix A.6. The key idea is to reduce our*
315 *UCB-based graph learning problem to a classical combinatorial multi-armed bandit (CMAB)*
316 *problem, enabling the application of standard analysis techniques.*

318 6 Experiments

319 To further demonstrate the effectiveness of our method, we conduct experiments comparing
320 our TGL-UCB approach with the classical imitation learning baseline, behavior cloning. The
321 details are presented below.

322 **Environment.** All experiments use a 4×4 FROZEN LAKE environment [Brockman et al.,
323 2016] modified to make *holes* passable (they yield a reward -1 but do not terminate the
324 episode) and to force the agent to move out of the hole in the next step. The goal square
325 returns $+1$ and ends the episode; otherwise the horizon is $H = 10$. On every step the intended
326 action is replaced by a uniformly-random valid action with probability 0.10. Observations
327 are one-hot state vectors and actions are the four cardinal moves.

328 **Demonstrations.** For each condition in Table 1, we build a set $\mathcal{D} = \{\tau_i\}$ of “expert”
329 trajectories that serve as offline data: these trajectories are produced by the deterministic
330 expert or by its stochastic variant. By our definition of conflicts, the trajectories produced
331 by any deterministic agent are not conflicting, which is why we decided to mix in some
332 trajectories generated by a stochastic agent. We then tested various trajectory set sizes
333 (from 1 to 20) and various generation settings (combinations of deterministic or stochastic
334 generation). No further interaction with the environment is allowed during training.

Table 1: Trajectory match probability (\uparrow better) over 10,000 episodes. Each row shows how many deterministic and stochastic demos are in \mathcal{D} .

Expert-Labeled Set	TGL-UCB (Ours)	BC	PPO Expert
15 det. & 5 stoch.	0.794	0.775	0.778
10 det. & 10 stoch.	0.801	0.778	0.800
5 det. & 15 stoch.	0.801	0.771	0.793
10 det. & 5 stoch.	0.814	0.793	0.803
8 det. & 7 stoch.	0.810	0.774	0.801
5 det. & 10 stoch.	0.779	0.758	0.780
10 det. only	0.778	0.753	0.775
5 det. & 5 stoch.	0.740	0.713	0.735
10 stoch. only	0.738	0.714	0.739
5 det. only	0.730	0.703	0.713
5 stoch. only	0.675	0.673	0.661
3 det. only	0.672	0.672	0.670
3 stoch. only	0.657	0.653	0.648
1 det. only	0.634	0.627	0.621

Methods compared.

- **TGL (ours).** Algorithm 2 (TGL-UCB) is run on \mathcal{D} with initial number of samples $m_0 = 10$, and successful probability $\delta = 0.9$. We solve for the exact MWIS solution in the algorithm (since our environment is small). The resulting maximum-weight independent set \mathcal{S}^+ is turned into a time-indexed lookup policy as described in Section 5 of the paper.
- **Behavioural Cloning (BC).** We train the supervised learner from `imitation.algorithms.bc` (batch size 8, 20 epochs) on the same trajectory set.
- **PPO Expert.** The reference expert policy used to generate the deterministic demonstrations (trained for 50 k steps with PPO).

Metric. Because numerical rewards are absent during training, we evaluate policies by the probability that a rollout $\hat{\tau}$ *exactly matches* one of the demonstration trajectories:

$$P_{\text{match}}(\pi) = \Pr_{\hat{\tau} \sim \pi} [\hat{\tau} \in \mathcal{D}].$$

For each policy we execute 10 000 episodes with fixed seed and report the empirical match frequency.

Discussion. Across every demo composition our TGL-UCB method outperforms or matches the behavioural-cloning baseline and usually surpasses the original PPO expert, despite never seeing the reward. The advantage is most pronounced when demonstrations are scarce or highly mixed, highlighting the benefit of directly learning from trajectories.

Limitations and Future Work. Our experiment was done using a single 4×4 FROZEN LAKE gridworld-like environment. More experiments can be done on larger or continuous MDPs. TGL-UCB is also more computationally expensive than Behavioral Cloning; we therefore capped the number of iterations at 50 (which is sufficient for the 4×4 FROZEN LAKE environment. Future work includes improving the computational efficiency of our methods.

7 Conclusion

This work introduces *Trajectory Graph Learning* (TGL), a novel framework for trajectory-level policy alignment that bypasses reward modeling. By leveraging structural assumptions often met in practice, TGL enables efficient and theoretically grounded planning in both known and unknown environments. Our theoretical results highlight the inherent complexity of direct trajectory imitation, while our algorithms demonstrate strong empirical gains over conventional imitation learning methods. These findings underscore the promise of structure-aware trajectory alignment for reliable long-horizon decision-making in reinforcement learning.

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A Technical Appendices and Supplementary Material

A.1 Remaining Algorithm pseudocodes

We provide the remaining algorithms in this section.

Algorithm 3 ENUMMWIS(G, w, ϵ_0)

Require: Conflict graph $G = (V, E)$ with $|V| = M$; weights $w : V \rightarrow (0, 1]$ satisfying $w(v) \geq \epsilon_0$
Ensure: Maximum-weight independent set S^*

- 1: $K \leftarrow \lfloor 1/\epsilon_0 \rfloor$ \triangleright any independent set has size $\leq K$
- 2: $W_{\text{best}} \leftarrow 0, S^* \leftarrow \emptyset$
- 3: **for all** subsets $X \subseteq V$ with $|X| \leq K$ **do**
- 4: **if** X is independent in G **then**
- 5: $W \leftarrow \sum_{v \in X} w(v)$
- 6: **if** $W > W_{\text{best}}$ **then**
- 7: $W_{\text{best}} \leftarrow W, S^* \leftarrow X$
- 8: **return** S^*

Algorithm 4 TGL-PRUNEDTREE

Require: Finite-horizon Tree MDP $(\mathcal{S}, \mathcal{A}, P, H)$; Expert-Labeled Trajectory Set $\mathcal{T}_{\text{sel}} = \{\tau_i\}_{i=1}^M$ where $\tau_i = (s_1^i, a_1^i, \dots, s_H^i, a_H^i)$.
Ensure: Pruned set \mathcal{M} with final weights

- 1: **for all** $\tau_i \in \mathcal{T}_{\text{sel}}$ **do**
- 2: $w_i \leftarrow \prod_{h=1}^{H-1} P(s_{h+1}^i | s_h^i, a_h^i)$
- 3: $\mathcal{M} \leftarrow \mathcal{T} = \{(\tau_i, w_i)\}_{i=1}^M, \{(\tau_{\text{com}}, w_{\text{com}})\} \leftarrow \{(\tau_i, w_i)\}$ \triangleright active trajectories
- 4: **for** $h = H, H-1, \dots, 1$ **do** \triangleright leaf \rightarrow root
- 5: $\text{Agg}[(s, a)] \leftarrow 0, \forall (s, a)$ \triangleright map $(s, a) \mapsto$ summed weight
- 6: $\text{Com}[(s, a)] \leftarrow \emptyset, \forall (s, a)$ \triangleright Combine the non-conflicting trajectories
- 7: **for each** $(\tau_{\text{com}}, w_{\text{com}}) \in \mathcal{M}$ **do**
- 8: Pick any trajectory $\tau \in \tau_{\text{com}}$ \triangleright Just choose a representative
- 9: $(s, a) \leftarrow (s_h(\tau), a_h(\tau))$
- 10: $\text{Agg}[(s, a)] \leftarrow \text{Agg}[(s, a)] + w_{\text{com}}$ \triangleright Aggregate the weights for the same (s_h, a_h)
- 11: $\text{Com}[(s, a)] \leftarrow \tau_{\text{com}} \cup \text{Com}[(s, a)]$
- 12: $\mathcal{M} \leftarrow \{(\text{Com}[(s, a)], \text{Agg}[(s, a)]) | \text{Com}[(s, a)] \neq \emptyset\}$
- 13: **for each** $s \in \{s | \text{Com}[(s, a)] \neq \emptyset\}$ **do**
- 14: $a^* \leftarrow \max_{a \in \mathcal{A}} \text{Agg}[(s, a)]$
- 15: Delete $\text{Com}[(s, a)], a \neq a^*$ from \mathcal{M}
- 16: $w^* \leftarrow \text{Agg}[(s, a)]$
- 17: **for all** $\tau_i \in \mathcal{M}$ with $\tau_i = (s_1^i, a_1^i, \dots, s_H^i, a_H^i)$ **do**
- 18: **for** $h = 1$ **to** H **do**
- 19: $\pi_h(s_h^i) \leftarrow a_h^i$
- 20: For states not covered by $\bigcup_{v_i \in \mathcal{M}} \tau_i$, set π_h via a default rule
- 21: **return** \mathcal{M}, π

836 **A.2 Example: BC Fails to Capture Expert Trajectories**

837 **MDP Setup**

- 838 • **States:** $S = \{s_1, s_2\}$
839 • **Actions:** $A = \{a_1, a_2\}$
840 • **Transitions:** Deterministic transitions from s_1 to s_2 with any action

841 **Expert Trajectories**

- 842 • $\tau_1 = (s_1, a_1, s_2, a_2)$
843 • $\tau_2 = (s_1, a_2, s_2, a_1)$

844 **Behavior Cloning (BC) Limitation** Behavior cloning only observes state-action pairs:

$$(s_1, a_1), (s_1, a_2), (s_2, a_1), (s_2, a_2)$$

845 resulting in:

- 846 • At s_1 , both a_1 and a_2 appear equally good.

847 • At s_2 , both a_1 and a_2 appear equally good.

848 Thus, BC will learn:

$$\pi(a_1|s_1) = \pi(a_2|s_1) = 0.5, \quad \pi(a_1|s_2) = \pi(a_2|s_2) = 0.5$$

849 **Consequence** Due to this ambiguity, the probability of correctly reproducing either expert
850 trajectory is:

$$P(\tau_1) = \pi(a_1|s_1) \cdot \pi(a_2|s_2) = 0.5 \times 0.5 = 0.25$$

851

$$P(\tau_2) = \pi(a_2|s_1) \cdot \pi(a_1|s_2) = 0.5 \times 0.5 = 0.25$$

852

$$\text{Total expert trajectory probability} = 0.5$$

853 **Direct Trajectory Alignment Advantage** Direct trajectory alignment explicitly optimizes
854 for the sequence:

$$\max_{\pi} \sum_{\tau \in \{\tau_1, \tau_2\}} P_{\pi}(\tau)$$

855 ensuring the policy preserves the correct sequence with probability 1, with either $\pi(a_1|s_1) =$
856 $1, \pi(a_2|s_2) = 1$; or $\pi(a_2|s_1) = 1, \pi(a_1|s_2) = 1$, since it learns to produce both trajectories as
857 whole entities, rather than decomposing them into ambiguous state-action pairs.

858 A.3 Proof of Theorem 1

859 In this section, we want to prove that finding the optimal policy in the binary-labeled
860 expert-labeled trajectory set setting is NP-complete. Before providing the main content of
861 proof. We first define the following variant of MWIS problem.

862 **Definition 6** (MWIS_{<1}).

863 **Input**

- 864 • A graph $G = (V, E)$;
- 865 • A weight function $w : V \rightarrow (0, 1]$;
- 866 • A rational threshold $K \in (0, 1)$.

867 **Condition** For every independent set $I \subseteq V$ it holds $\sum_{v \in I} w(v) < 1$.

868 **Question** Does there exist an independent set $I \subseteq V$ with $\sum_{v \in I} w(v) \geq K$?

869 **Theorem 5** (MWIS_{<1} is NP-complete). The decision problem in Definition 6 is NP-
870 complete.

871 **Proof. Membership in NP.** A certificate is an independent set $I \subseteq V$. We can verify
872 independence in $O(|E|)$ time and compute $\sum_{v \in I} w(v)$ in $O(|I|)$ time, so the problem lies
873 in NP.

874 We reduce from the classical MAXIMUM INDEPENDENT SET (MIS) problem, known to be
875 NP-complete. The problem is: A graph $G = (V, E)$ and an integer $t \geq 1$, and the question
876 is: does G contain an independent set of size at least t ?

877 **Reduction.** Given an instance (G, t) of MIS with $n := |V(G)|$, construct (G, w, K) for
878 MWIS_{<1} as follows

$$w(v) := \frac{1}{n+1} \quad \forall v \in V(G), \quad K := \frac{t}{n+1} (< 1).$$

879 The construction uses only $O(\log n)$ -bit rationals, hence is polynomial in n .

880 For any independent set I we have

$$\sum_{v \in I} w(v) = \frac{|I|}{n+1} \leq \frac{n}{n+1} < 1,$$

881 so the promise in Definition 6 is satisfied.

Because every vertex has the same weight,

$$|I| \geq t \iff \sum_{v \in I} w(v) = \frac{|I|}{n+1} \geq \frac{t}{n+1} = K.$$

Thus $(G, t) \in \text{MIS}$ iff $(G, w, K) \in \text{MWIS}_{<1}$.

The reduction is polynomial, establishing NP-hardness. Since $\text{MWIS}_{<1}$ is in NP and NP-hard, it is NP-complete. \square

Now we can establish the proof of Theorem 1.

Proof. First, let us restate the original problem.

Original Problem: Find the optimal deterministic policy

Input: A MDP and the binary expert-labeled trajectory set $\mathcal{T}_{\text{sel}} = \{\tau_i\}_{i=1}^M$ (or called level-1 trajectory)

Question: Is there any deterministic policy that can visit the level-1 trajectories with probability at least p ?

We will do this in two steps:

Step 1: Show the original problem is in NP

To show that the original problem is in NP, we must demonstrate that a given solution can be verified in polynomial time.

Given a deterministic policy $\pi = \{\pi_h\}_{h \in [H]}$, we can:

1. **Check whether the good trajectory does not have conflict with policy π :**
To be specific, for a good trajectory $\tau = (s_1, a_1, s_2, a_2, \dots, s_H, a_H)$, check whether $\pi_1(s_1) = a_1, \pi_2(s_2) = a_2, \dots, \pi_H(s_H) = a_H$. If yes, add up the weight (the probability product) of this trajectory (We denote this set as Q). The complexity is at most $O(M \times H)$.
2. **Sum the weights:** Calculate $\sum_{w \in Q} w$ and check if it is at least p . This takes at most $O(M)$ time.

Since both checks can be performed in polynomial time, the original problem is in NP.

Step 2: Reduction from $\text{MWIS}_{<1}$

We will reduce from the $\text{MWIS}_{<1}$ problem (Definition 6), which is known to be NP-complete from Theorem 2.

Reduction from $\text{MWIS}_{<1}$ to original problem

1. **Any weighted graph can be represented as a subset of trajectories with corresponding probability** The basic idea is because we can always find a MDP with sufficient trajectories to represent the relation between these M vertexes, i.e. $M \ll O(|S|^H |A|^H)$. The construction process is detailed in Algorithm 5 and 6. To be specific, for an arbitrary graph, we can enumerate every vertex in a Breadth-First Search (**BFS**). Each time we process one vertex and use conflict trajectory pairs to record all the edges it connects to. Then we get a set of trajectories that encoded the information of this graph. The second step is to assign the probability of the MDP given the weights of the graph, which is explained in Algorithm 6. The basic idea is to assign the probability kernel with the given weights $w(v) \in (0, 1]$, and make sure the summation of all probability odds is still 1.
2. **Good policy implies high weight independent set.** First, for any deterministic policy π , it will produce a set of non-conflicting trajectories (any two of these trajectories are non-conflicting), we denote this set as $\Sigma(\pi)$, and now if a deterministic policy π that can visit the level-1 trajectories with probability at least p , then $\sum_{w \in \Sigma(\pi)} w > p$, which implies there exists an independent set that the summation of weights is larger than p .

Notice that the reduction process in Algorithm 5 and 6 is polynomial in M , therefore, we can conclude the original problem is also NP-complete.

Algorithm 5 BFS-Based Trajectory Construction

Require: Graph $G = (V, E)$, horizon H **Ensure:** Trajectories $\{\tau(v)\}_{v \in V}$, each of length H

```
1: Pick an arbitrary root vertex  $v_1 \in V$ 
2: Fix  $\tau(v_1) \leftarrow (s_1, a_1, \dots, s_H, a_H)$ 
3: Visited  $\leftarrow \{v_1\}$ ; enqueue  $v_1$  in queue  $Q$ 
4: for all  $v \in V \setminus \{v_1\}$  do
5:    $\tau(v) \leftarrow (s_1, a_1, s_U, a_U, \dots, s_U, a_U)$  ▷ undecided after step 1
6: while  $Q$  not empty do
7:    $u \leftarrow \text{Dequeue}(Q)$ 
8:   for all neighbor  $v$  of  $u$  with  $v \notin \text{Visited}$  do
9:     Fix remaining undecided steps in  $\tau(u)$  so it is unique
10:    Construct  $\tau(v)$  so that there exists a time  $t$  with  $s_t^{\tau(v)} = s_t^{\tau(u)}$  and  $a_t^{\tau(v)} \neq a_t^{\tau(u)}$ 
11:    Visited  $\leftarrow \text{Visited} \cup \{v\}$ ; enqueue  $v$ 
12: return  $\{\tau(v)\}_{v \in V}$ 
```

Algorithm 6 Transition-Probability Assignment Using Vertex Weights

Require: Graph $G = (V, E)$ with weights $w : V \rightarrow (0, 1]$, $\sum_{v \in S} w(v) \leq 1$, S is an independent set.**Require:** Trajectories $\{\tau(v)\}_{v \in V}$ from Alg. 5**Ensure:** Transition kernel $P(\cdot | \cdot, \cdot)$

```
/* Preparation */
1: for all  $v \in V$  do
2:   Write  $\tau(v) = (s_1, a_1, \dots, s_H, a_H)$ 
/* First state-action pair  $(s_1, a_1)$  */
3: for all edge  $\{v_1, v_2\} \in E$  do
4:    $P(s_2 | s_1, a_1) \leftarrow \max\{w(v_1), w(v_2)\}$ 
5:   Distribute remaining mass over other successors of  $(s_1, a_1)$ 
/* Finalize each adjacent pair */
6: for all edge  $(u, v) \in E$  do
7:   if  $w(u) > w(v)$  then
8:     Make the remainder of  $\tau(u)$  deterministic
9:     In  $\tau(v)$  add one stochastic step  $t$  with prob.  $w(v)/w(u)$  where  $s_t^{\tau(v)} = s_t^{\tau(u)}$  and  $a_t^{\tau(v)} \neq a_t^{\tau(u)}$ 
10:  else
11:    (Symmetric update with  $u \leftrightarrow v$ )
12: for all  $(s, a)$  with stochastic successors do
13:   Normalize  $P(\cdot | s, a)$  so  $\sum_x P(x | s, a) = 1$ 
14: return  $P(\cdot | \cdot, \cdot)$ 
```

928 A.4 Proof of Theorem 2

929 **Time complexity of Algorithm 1 with oracle Algorithm 3.** Let $M = |\mathcal{T}_{\text{sel}}|$ and H the
930 horizon length, and set

$$K = \left\lfloor \frac{1}{\epsilon_0} \right\rfloor.$$

931 Then

- 932 1. ****Conflict-graph construction**** (lines 3–7): $O\left(\binom{M}{2} \cdot H\right) = O(M^2 H)$ time.
- 933 2. ****Weight computation**** (lines 9–11): $O(MH)$.
- 934 3. ****Oracle call**** ENUMMWIS(G, w, ϵ_0) (Alg. 3):

$$\sum_{i=0}^K \binom{M}{i} \cdot O(i^2) = O\left(\sum_{i=0}^K \binom{M}{i}\right) = O(M^K) \quad (\text{since } K \text{ is a constant}).$$

- 935 4. ****Policy extraction**** (lines 15–18): $O(|S| H) = O(KH)$.

936 Putting these together gives

$$T(M, H) = O(M^2 H + MH + M^K + KH) = O(M^2 H + M^K).$$

937 In particular, if ϵ_0 (hence K) is a fixed constant, this is polynomial time $O(M^2 H + M^{1/\epsilon_0})$.

938 A.5 Proof of Theorem 3

939 *Proof of Time Complexity.* Let:

- 940 • M be the number of trajectories,
- 941 • H be the trajectory horizon,
- 942 • $|\mathcal{A}|$ be the number of discrete actions,
- 943 • $|\mathcal{S}|$ be the number of possible states per timestep.

944 **1. Weight Computation:** Each trajectory τ_i is assigned a weight via the product of $H - 1$
945 transition probabilities. Across M trajectories, this requires:

$$\mathcal{O}(M \cdot H)$$

946 **2. Backward Pruning Loop:** For each timestep $h = H, H - 1, \dots, 1$ (total H iterations):

- 947 • Aggregating weights across identical (s_h, a_h) pairs requires scanning all active trajectories:
948 $\mathcal{O}(M)$,
- 949 • Merging trajectories that share the same (s_h, a_h) pair: $\mathcal{O}(M)$,
- 950 • For each state s_h , selecting the action a_h with the maximum total weight among at most
951 $|\mathcal{A}|$ options leads to: $\mathcal{O}(M \cdot |\mathcal{A}|)$ in the worst case (since there are at most M unique
952 state-action pairs).

953 Therefore, the total cost per timestep is:

$$\mathcal{O}(M \cdot |\mathcal{A}|)$$

954 and over H timesteps:

$$\mathcal{O}(H \cdot M \cdot |\mathcal{A}|)$$

955 **3. Final Output:** The pruned result is returned by traversing at most M remaining
956 elements:

$$\mathcal{O}(M)$$

957 **Total Time Complexity:** Summing all terms, the dominating component is from the
958 backward pruning loop, yielding the final result:

$$\boxed{\mathcal{O}(H \cdot M \cdot |\mathcal{A}|)}$$

959

□

960 A.6 Proof of Theorem 4

961 *Proof.* The proof of Theorem 4 is based on the Theorem 1 in [Chen et al., 2013]. For the
 962 convenience of the readers, we show the complete process here.

963 For variable T_i , let $T_{i,t}$ be the value of T_i at the end of round t , that is, $T_{i,t}$ is the number of
 964 times policy played that matches trajectory v_i in the first t rounds. For variable $\hat{\mu}_i$, let $\hat{\mu}_{i,s}$
 965 be the value of $\hat{\mu}_i$ after the policy which matched trajectory v_i is played s times. Then, the
 966 value of variable $\hat{\mu}_i$ at the end of round t is $\hat{\mu}_{i,T_{i,t}}$. For variable u_i , let $u_{i,t}$ be the value of
 967 u_i at the end of round t . Let $\bar{\mathbf{u}}_t = (u_{1,t}, \dots, u_{M,t})$ be the random vector fed to the MWIS
 968 oracle as the input in line 12 of Algorithm 2 at round t .

969 We also maintain counter Q_i for each trajectory v_i after the M initialization rounds. Let $Q_{i,t}$
 970 be the value of Q_i after the t -th round and $Q_{i,M} = 1$. Note that $\sum_i Q_{i,M} = M$. Counters
 971 $\{Q_i\}_{i=1}^M$ are updated as follows.

972 For a round $t > M$, let S_t be the independent set selected in round t by the MWIS oracle
 973 (line 12 of Algorithm 2). Round t is bad if the oracle selects a bad set $S_t \in \mathcal{S}_{\text{sub}}$. If round t
 974 is bad, let $i = \text{argmin}_{j \in S_t} Q_{j,t-1}$. We increment Q_i by one, i.e., $Q_{i,t} = Q_{i,t-1} + 1$. That is,
 975 we find the trajectory v_i with the smallest counter in S_t and increment its counter. If i is
 976 not unique, we pick an arbitrary arm with the smallest counter in S_t . On the other hand, if
 977 $S_t \notin \mathcal{S}_B$, no counter will be incremented.

978 By definition $Q_{i,t} \leq T_{i,t}$. Notice that in every bad round, exactly one counter in $\{Q_i\}_{i=1}^M$ is
 979 incremented, so the total number of bad rounds in the first n rounds is less than or equal to
 980 $\sum_i Q_{i,n}$.

Define $\ell_t = \frac{6M^2 \ln t}{\Delta_{\min}^2}$. Consider a bad round $t, S_t \in \mathcal{S}_{\text{sub}}$ is selected and counter Q_i of some
 arm $i \in S_t$ is updated. We have

$$\begin{aligned}
 & \sum_{i=1}^m Q_{i,n} - m \cdot (\ell_n + 1) = \sum_{t=m+1}^n \mathbb{I}\{S_t \in \mathcal{S}_{\text{sub}}\} - m\ell_n \\
 & \leq \sum_{t=m+1}^n \sum_{i \in [m]} \mathbb{I}\{S_t \in \mathcal{S}_{\text{sub}}, Q_{i,t} > Q_{i,t-1}, Q_{i,t-1} > \ell_n\} \\
 & \leq \sum_{t=m+1}^n \sum_{i \in [m]} \mathbb{I}\{S_t \in \mathcal{S}_{\text{sub}}, Q_{i,t} > Q_{i,t-1}, Q_{i,t-1} > \ell_t\} \\
 & = \sum_{t=m+1}^n \mathbb{I}\{S_t \in \mathcal{S}_{\text{sub}}, \forall i \in S_t, Q_{i,t-1} > \ell_t\} \\
 & \leq \sum_{t=m+1}^n \mathbb{I}\{S_t \in \mathcal{S}_{\text{sub}}, \forall i \in S_t, T_{i,t-1} > \ell_t\}
 \end{aligned}$$

981 We first claim that $\Pr(\{S_t \in \mathcal{S}_{\text{sub}}, \forall i \in S_t, T_{i,t-1} > \ell_t\}) \leq 2 \cdot M \cdot t^{-2}$.

In fact, for any $i \in [M]$,

$$\begin{aligned}
 & \Pr \left[|\hat{\mu}_{i,T_{i,t-1}} - \mu_i| \geq \sqrt{3 \ln t / (2T_{i,t-1})} \right] \\
 & = \sum_{s=1}^{t-1} \Pr \left[\left\{ |\hat{\mu}_{i,s} - \mu_i| \geq \sqrt{3 \ln t / (2s)}, T_{i,t-1} = s \right\} \right] \\
 & \leq \sum_{s=1}^{t-1} \Pr \left[|\hat{\mu}_{i,s} - \mu_i| \geq \sqrt{3 \ln t / (2s)} \right] \\
 & \leq t \cdot 2e^{-3 \ln t} = 2t^{-2}
 \end{aligned}$$

982 where the last inequality is due to the Chernoff Hoeffding bound. Define $\Lambda_{i,t} =$
 983 $\sqrt{\frac{3 \ln t}{2T_{i,t-1}}}$ (a random variable since $T_{i,t-1}$ is a random variable), and event $E_t =$
 984 $\{\forall i \in [m], |\hat{\mu}_{i,T_{i,t-1}} - \mu_i| \leq \Lambda_{i,t}\}$. By union bound, $\Pr[\neg E_t] \leq 2 \cdot M \cdot t^{-2}$. According

985 to line 11 of Algorithm 2, we have $u_{i,t} - \hat{\mu}_{i,T_{i,t-1}} = \Lambda_{i,t}$. Thus $|\hat{\mu}_{i,T_{i,t-1}} - \mu_i| \leq \Lambda_{i,t}$ implies
 986 that $u_{i,t} \geq \mu_i$.

Let $\Lambda = \sqrt{\frac{3 \ln t}{2 \ell_t}}$, which is not a random variable. Define random variable $\Lambda_t = \max \{\Lambda_{i,t} \mid i \in S_t\}$. Then

$$E_t \Rightarrow \forall i \in S_t, |u_{i,t} - \mu_i| \leq 2\Lambda_t \\ \{S_t \in \mathcal{S}_{\text{sub}}, \forall i \in S_t, T_{i,t-1} > \ell_t\} \Rightarrow \Lambda > \Lambda_t$$

Let $\bar{\mathbf{u}}_t = (u_{1,t}, \dots, u_{M,t})$ be the vector representing the adjusted expectation vector at round t . Then,

$$E_t \Rightarrow \bar{\mathbf{u}}_t \geq \boldsymbol{\mu}$$

If $\{E_t, S_t \in \mathcal{S}_{\text{sub}}, \forall i \in S_t, T_{i,t-1} > \ell_t\}$ holds at time t , we have the following important derivation:

$$\sum_{v \in S_t} p_v + 2M\Lambda > \sum_{v \in S_t} p_v + 2M\Lambda_t \geq \sum_{v \in S_t} u_v \geq p^*$$

Since $\ell_t = \frac{6M^2 \ln t}{\Delta_{\min}^2}$, we have $2M\Lambda = \Delta_{\min}$. Therefore,

$$\Pr[\{E_t, S_t \in \mathcal{S}_{\text{sub}}, \forall i \in S_t, T_{i,t-1} > \ell_t\}] = 0 \Rightarrow \\ \Pr[\{S_t \in \mathcal{S}_{\text{sub}}, \forall i \in S_t, T_{i,t-1} > \ell_t\}] \\ \leq \Pr[\neg E_t] \leq 2 \cdot M \cdot t^{-2}.$$

987 The claim thus holds. We have, $\mathbb{E} \left[\sum_{i=1}^M Q_{i,n} \right] \leq M(\ell_n + 1) + \sum_{t=1}^n \frac{2M}{t^2} \leq \frac{6M^3 \ln n}{\Delta_{\min}^2} +$
 988 $\left(\frac{\pi^2}{3} + 1 \right) \cdot M$.

Notice that each time we select a bad independent set at time t , we incur a regret at most Δ_{\max} . Then we obtain the regret bound as follows.

$$\text{Regret}(n) \\ \leq \mathbb{E} \left[\sum_{i=1}^M Q_{i,n} \right] \cdot \Delta_{\max} \\ \leq \left(\frac{6 \log n \cdot M^2}{\Delta_{\min}^2} + \frac{\pi^2}{3} + 1 \right) \cdot M \cdot \Delta_{\max}^2 \\ = O \left(\frac{M^3 \log K \cdot \Delta_{\max}}{\Delta_{\min}^2} \right)$$

989

□

990 A.7 Experimental Details and Hyperparameters

991 All hyper-parameters, stopping criteria and environment settings are listed in Table 2.

992 A.8 Compute Resources

993 All runs use a single Google Colab instance with an NVIDIA T4 (16 GB GPU RAM) kernel.
 994 Training the PPO expert for 50 000 steps takes around 1 min 20 s; one TGL-UCB run
 995 (50 iterations, 20 nodes) takes around 45 s. The evaluation of the three methods that we
 996 compare (TGL, BC, and PPO expert) takes around 1 min 30 s. Most of the code are not
 997 GPU accelerated, so using a CPU kernel is also feasible.

Table 2: Hyper-parameters used in every experiment.

Component	Hyper-parameter	Value
<i>Environment</i>		
	Grid size	4×4
	Episode horizon H	10
	Random-action noise	0.10
<i>PPO expert</i>		
	Learning rate	1×10^{-3}
	n_steps	128
	Batch size	64
	Epochs per update	4
	Discount γ	0.99
	GAE λ	0.95
	Clip range	0.2
	Total steps	50 000
<i>TGL-UCB</i>		
	δ	0.9
	Initial samples m_0	10
	ϵ	0.01
	Max iterations	50
<i>Behavioural Cloning</i>		
	Batch size	8
	Training epochs	20